Calculus Warm Up #11-5

Evaluate:

$$\int_0^{\pi/3} \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$$

Staple and turn in:

Warm Up on top

Classwork: p. 526, # 5 - 8

HW Questions: p. 536

3. $\int \frac{3}{x^2 + x - 2}\, dx$

5. $\int \frac{5 - x}{2x^2 + x - 1}\, dx$

7. $\int \frac{x^2 + 12x + 12}{x^3 - 4x}\, dx$

9. $\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\, dx$

11. $\int \frac{4x^2 + 2x - 1}{x^3 + x^2}\, dx$

$x^2(x+1)$

$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

Basic: $4x^2 + 2x - 1 = Ax(x+1) + B(x+1) + Cx^2$

15. $\int \frac{3x}{x^2 - 6x + 9}\, dx = \frac{A}{x-3} + \frac{B}{(x-3)^2}$

$(x-3)^2$

27) $\int \frac{}{(x-2)(x^2+4)}\, dx$

$\frac{A}{x-2} + \frac{Bx+C}{x^2+4}$

Basic Eq:

$6x^2 - 3x + 14 = A(x^2+4) + (Bx+C)(x-2)$

31)

9.7 Improper Integrals

Remember: Definition of a Definite Integral

$\int_a^b f(x)\,dx$ f(x) must be continuous on [a, b]

Improper Integrals

If a &/or b = $\pm\infty$

Or if

f(x) has any infinite discontinuities on [a, b]

Infinite Discontinuities when:

$\lim_{x \to c} f(x) = \pm\infty$ from the right or left

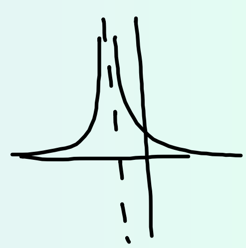
Examples

$\int_{-1}^{\infty} \frac{1}{x}\,dx$; $\int_1^5 \frac{1}{\sqrt{x-1}}\,dx$; $\int_{-2}^{2} \frac{1}{(x+1)^2}\,dx$

undef @ $x=0$
(∞ discontinuity)
AND upper limit ∞

$\lim_{x\to 1^+} = \frac{1}{0} = \infty$

$\lim_{x\to -1} = \infty$



Evaluating Improper Integrals

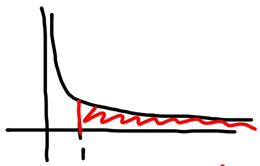
Use a limit process:

$$\int_1^\infty \frac{1}{x^2}\,dx \qquad \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{b}+1\right]$$

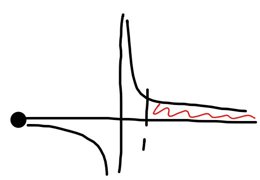
$$= -\frac{1}{\infty}\to 0 + 1$$

$$= \boxed{1}$$



The integral = area under the curve is converging to $\boxed{1}$

$$\int_1^\infty \frac{1}{x}\,dx$$



$$= \lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty}\Big[\ln|x|\Big]_1^b = \lim_{b\to\infty}\left[\ln b - \ln 1\right]$$

$\ln 1 = 0$

$$= \ln \infty$$

$$= \infty \quad \text{DNE}$$

☆ The area is infinite. The integral does not converge. Diverges.

$$\int_{-1}^{2} \frac{1}{x^3}\,dx$$

Infinite discontinuity at x=0. Split the integral and use the limit process to approach the discontinuity from both sides.

$$= \lim_{b \to 0^-} \int_{-1}^{b} \frac{1}{x^3}\,dx + \lim_{a \to 0^+} \int_{a}^{2} \frac{1}{x^3}\,dx$$

$$= \lim_{b \to 0^-} \left[-\frac{1}{2x^2}\right]_{-1}^{b} + \lim_{a \to 0^+} \left[-\frac{1}{2x^2}\right]_{a}^{2}$$

$$= \lim_{b \to 0^-} \left[-\frac{1}{2b^2} + \frac{1}{2}\right] + \lim_{a \to 0^+} \left[-\frac{1}{8} + \infty\right]$$

$$\left(-\infty + \frac{1}{2}\right) + \left(-\frac{1}{8} + \infty\right)$$

☆ Consider each part separately. If either of the improper integrals diverges, the the original integral diverges.

<u>Next week's agenda:</u>

Mon: Finish 9.7

Tues: HW Quiz, pgs. 516, 526, 536, 553

Review (Pink WS turned in Friday)

Wed: Review (Tan WS turned in Friday)

Thur: Test Part I, no calculator

HW: final exam review

Fri: Test Part 2, with calculator

HW: final exam review

HW: p. 553, # 1 - 7 all

Determine if it converges or diverges,

evaluate if it converges.